

R-Parity in Supersymmetric Left-Right Models.

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On this article we show explicitly that Supersymmetric Left-Right Models already satisfy the *R*-parity. They also respect *L*-parity and *B*-parity.

1. Introduction

Although the Standard Model (SM) gives very good results in explaining the observed properties of the charged fermions, it is unlikely to be the ultimate theory. It maintains the masslessness of the neutrinos to all orders in perturbation theory, and even after non-perturbative effects are included. The recent groundbreaking discovery of nonzero neutrino masses and oscillations ¹ has put massive neutrinos as one of evidences on physics beyond the SM.

The Super-Kamiokande experiments on the atmospheric neutrino oscillations have indicated to the difference of the squared masses and the mixing angle with fair accuracy ^{2,3}

$$\Delta m_{\text{atm}}^2 = 1.3 \div 3.0 \times 10^{-3} \text{eV}^2, \quad (1)$$

$$\sin^2 2\theta_{\text{atm}} > 0.9. \quad (2)$$

While, those from the combined fit of the solar and reactor neutrino data point to

$$\Delta m_{\odot}^2 = 8.0_{-0.4}^{+0.6} \times 10^{-5} \text{eV}^2, \quad (3)$$

$$\tan^2 \theta_{\odot} = 0.45_{-0.07}^{+0.09}. \quad (4)$$

Since the data provide only the information about the differences in m_{ν}^2 , the neutrino mass pattern can be either almost degenerate or hierarchical. Among the hierarchical possibilities, there are two types of normal and inverted hierarchies. In the literature, most of the cases explore normal hierarchical one in each. In this paper, we will mention on a supersymmetric model which naturally gives rise to three pseudo-Dirac neutrinos with an inverted hierarchical mass pattern.

The gauge symmetry of the SM as well as those of many extensional models by themselves fix only the gauge bosons. The fermions and Higgs contents have to be chosen somewhat arbitrarily. In the SM, these choices are made in such a way that the neutrinos are massless as mentioned. However, there are other choices based on the SM symmetry that neutrinos become massive. We know these from the popular seesaw ⁴ and radiative ⁵ models.

Certainly a very popular extension of the SM is the left-right symmetric theories⁶, which attribute the observed parity asymmetry in the weak interactions to the spontaneous breakdown of Left-Right symmetry, i.e. generalized parity transformations. Furthermore, Left-Right symmetry plays an important role in attempting to understand the smallness of CP violation⁷.

Apart from its original motivation of providing a dynamic explanation for the parity violation observed in low-energy weak interactions, this model differs from the SM in another important aspect; it explains the observed lightness of neutrinos in a natural way and it can also solve the strong CP problem.

On the technical side, the left-right symmetric model has a problem similar to that in the SM: the masses of the fundamental Higgs scalars diverge quadratically. As in the SM, the SUSYLR can be used to stabilize the scalar masses and cure this hierarchy problem.

On the literature there are two different SUSYLR models. They differ in their $SU(2)_R$ breaking fields: one uses $SU(2)_R$ triplets (SUSYLRT) and the other $SU(2)_R$ doublets (SUSYLRD). Theoretical consequences of these models can be found in various papers including⁸ and⁹ respectively.

Another, maybe more important *raison d'être* for supersymmetric Left-Right models is the fact that they lead naturally to R-parity conservation. Namely, Left-Right models contain a $B - L$ gauge symmetry, which allows for this possibility¹⁰. All that is needed is that one uses a version of the theory that incorporates a see-saw mechanism¹¹ at the renormalizable level. More precisely, R-parity (which keeps particles invariant, and changes the sign of sparticles) can be written as

$$R = (-1)^{3(B-L)+2S} \quad (5)$$

where S is the spin of the particle. It can be shown that in these kind of theories, invariance under $B - L$ implies R-parity conservation¹⁰. It is just the goal of this article. We will show that both models, SUSYLRT and SUSYLRD, are invariant under R -Parity transformation. Before, we start it is useful to remember that the choice of the triplets is preferable to doublets because in the first case we can generate a large Majorana mass for the right-handed neutrinos¹².

The goal of this article is to show this appealing characteristics of having automatic R -parity conservation in both models SUSYLRT and SUSYLRD.

2. R -Symmetry

It is important to note that the SM can explain the conservation of lepton number (L) and of baryon number (B) without needing to any discrete symmetry. However, this is not the case of supersymmetric theories where only if interactions of conserving both L and B are required, one has to impose one discrete symmetry.

The R -symmetry was introduced in 1975 by A. Salam and J. Strathdee¹³ and in an independent way by P. Fayet¹⁴ to avoid the interactions that violate either lepton number or baryon number. There is very nice review about this subject in Refs.^{15,16,17}.

Superfield	Usual Particle	Spin	Superpartner	Spin
$\hat{V}' (U(1))$	B_m	1	\tilde{B}	$\frac{1}{2}$
$\hat{V}_L^i (SU(2)_L)$	W_{mL}^i	1	\tilde{W}_L^i	$\frac{1}{2}$
$\hat{V}_R^i (SU(2)_R)$	W_{mR}^i	1	\tilde{W}_R^i	$\frac{1}{2}$
$\hat{V}_c^a (SU(3))$	g_m^a	1	\tilde{g}^a	$\frac{1}{2}$
$\hat{Q}_i \sim (\mathbf{3}, \mathbf{2}, \mathbf{1}, 1/3)$	$(u_i, d_i)_{iL}$	$\frac{1}{2}$	$(\tilde{u}_{iL}, \tilde{d}_{iL})$	0
$\hat{Q}_i^c \sim (\mathbf{3}^*, \mathbf{1}, \mathbf{2}, -1/3)$	$(d_i^c, -u_i^c)_{iL}$	$\frac{1}{2}$	$(\tilde{d}_{iL}^c, -\tilde{u}_{iL}^c)$	0
$\hat{L}_a \sim (\mathbf{1}, \mathbf{2}, \mathbf{1}, -1)$	$(\nu_a, l_a)_{aL}$	$\frac{1}{2}$	$(\tilde{\nu}_{aL}, \tilde{l}_{aL})$	0
$\hat{L}_a^c \sim (\mathbf{1}, \mathbf{1}, \mathbf{2}, 1)$	$(l_a^c, -\nu_a^c)_{aL}$	$\frac{1}{2}$	$(\tilde{l}_{aL}^c, -\tilde{\nu}_{aL}^c)$	0
$\hat{\Delta}_L \sim (\mathbf{1}, \mathbf{3}, \mathbf{1}, 2)$	$\begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & \frac{-\delta_L^+}{\sqrt{2}} \end{pmatrix}$	0	$\begin{pmatrix} \frac{\tilde{\delta}_L^+}{\sqrt{2}} & \tilde{\delta}_L^{++} \\ \tilde{\delta}_L^0 & \frac{-\tilde{\delta}_L^+}{\sqrt{2}} \end{pmatrix}$	$\frac{1}{2}$
$\hat{\Delta}'_L \sim (\mathbf{1}, \mathbf{3}, \mathbf{1}, -2)$	$\begin{pmatrix} \frac{\delta_L'^-}{\sqrt{2}} & \delta_L'^0 \\ \delta_L'^-- & \frac{-\delta_L'^-}{\sqrt{2}} \end{pmatrix}$	0	$\begin{pmatrix} \frac{\tilde{\delta}_L'^-}{\sqrt{2}} & \tilde{\delta}_L'^0 \\ \tilde{\delta}_L'^-- & \frac{-\tilde{\delta}_L'^-}{\sqrt{2}} \end{pmatrix}$	$\frac{1}{2}$
$\hat{\delta}_L^c \sim (\mathbf{1}, \mathbf{1}, \mathbf{3}, -2)$	$\begin{pmatrix} \frac{\lambda_L^-}{\sqrt{2}} & \lambda_L^0 \\ \lambda_L^{--} & \frac{-\lambda_L^-}{\sqrt{2}} \end{pmatrix}$	0	$\begin{pmatrix} \frac{\tilde{\lambda}_L^-}{\sqrt{2}} & \tilde{\lambda}_L^0 \\ \tilde{\lambda}_L^{--} & \frac{-\tilde{\lambda}_L^-}{\sqrt{2}} \end{pmatrix}$	$\frac{1}{2}$
$\hat{\delta}_L'^c \sim (\mathbf{1}, \mathbf{1}, \mathbf{3}, 2)$	$\begin{pmatrix} \frac{\lambda_L'^+}{\sqrt{2}} & \lambda_L'^{++} \\ \lambda_L'^0 & \frac{-\lambda_L'^+}{\sqrt{2}} \end{pmatrix}$	0	$\begin{pmatrix} \frac{\tilde{\lambda}_L'^+}{\sqrt{2}} & \tilde{\lambda}_L'^{++} \\ \tilde{\lambda}_L'^0 & \frac{-\tilde{\lambda}_L'^+}{\sqrt{2}} \end{pmatrix}$	$\frac{1}{2}$
$\hat{\Phi} \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0)$	$\begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$	0	$\begin{pmatrix} \tilde{\phi}_1^0 & \tilde{\phi}_1^+ \\ \tilde{\phi}_2^- & \tilde{\phi}_2^0 \end{pmatrix}$	$\frac{1}{2}$
$\hat{\Phi}' \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0)$	$\begin{pmatrix} \chi_1^0 & \chi_1^+ \\ \chi_2^- & \chi_2^0 \end{pmatrix}$	0	$\begin{pmatrix} \tilde{\chi}_1^0 & \tilde{\chi}_1^+ \\ \tilde{\chi}_2^- & \tilde{\chi}_2^0 \end{pmatrix}$	$\frac{1}{2}$

2.1. Discrete *R*-Parity in *SUSYLRT*

The particle content of the model is given at Tab.(1) (for recent work see for example [?] and references therein). In parentheses it appears the transformation properties under the respective $(SU(3)_C, SU(2)_L, SU(2)_R, U(1)_{B-L})$. The Lagrangian is given at Ref. ¹⁸.

The Left-Right models may have doubly charged Scalars, see Tab.(1), as a consequence of this when we construct their supersymmetric version, we get double charged charginos ¹⁹.

The most general superpotential W ⁸ is given by

$$\begin{aligned}
W = & M_{\Delta} Tr(\hat{\Delta}_L \hat{\Delta}'_L) + M_{\delta^c} Tr(\hat{\delta}_L^c \hat{\delta}_L'^c) + \mu_1 Tr(\imath \tau_2 \hat{\Phi} \imath \tau_2 \hat{\Phi}) + \mu_2 Tr(\imath \tau_2 \hat{\Phi}' \imath \tau_2 \hat{\Phi}') \\
& + \mu_3 Tr(\imath \tau_2 \hat{\Phi} \imath \tau_2 \hat{\Phi}') + f_{ab} Tr(\hat{L}_a \imath \tau_2 \hat{\Delta}_L \hat{L}_b) + f_{ab}^c Tr(\hat{L}_a^c \imath \tau_2 \hat{\delta}_L^c \hat{L}_b^c) \\
& + h_{ab}^l Tr(\hat{L}_a \hat{\Phi} \imath \tau_2 \hat{L}_b^c) + \tilde{h}_{ab}^l Tr(\hat{L}_a \hat{\Phi}' \imath \tau_2 \hat{L}_b^c) + h_{ij}^q Tr(\hat{Q}_i \hat{\Phi} \imath \tau_2 \hat{Q}_j^c) \\
& + \tilde{h}_{ij}^q Tr(\hat{Q}_i \hat{\Phi}' \imath \tau_2 \hat{Q}_j^c) + W_{NR}.
\end{aligned} \tag{6}$$

Where h^l, \tilde{h}^l, h^q and \tilde{h}^q are the Yukawa couplings for the leptons and quarks, respectively, and f and f^c are the couplings for the triplets scalar bosons. We must emphasize that due to the conservation of $B - L$ symmetry, Δ'_L and $\delta_L'^c$ do not couple with the leptons and quarks. Here W_{NR} denotes (possible) non-renormalizable terms arising from higher scale physics or Planck scale effects²⁰. This model can be embedded in a supersymmetric grand unified theory as $SO(10)$ ²¹.

Applying the invariance conditions under R -parity transformation in Eq.(6) we get the following equations

$$\begin{aligned}
n_{\Delta_L} + n_{\Delta'_L} = 0, n_{\delta_L^c} + n_{\delta_L'^c} = 0, 2n_{\Phi} = 2n_{\Phi'} = 0, n_{\Phi} + n_{\Phi'} = 0, 2n_L + n_{\Delta_L} = 0, 2n_{L^c} + n_{\delta_L^c} = 0, \\
n_L + n_{L^c} + n_{\Phi} = 0, n_L + n_{L^c} + n_{\Phi'} = 0, n_Q + n_{Q^c} + n_{\Phi} = 0, n_Q + n_{Q^c} + n_{\Phi'} = 0.
\end{aligned} \tag{7}$$

For example, choosing

$$\begin{aligned}
n_{\Delta_L} = -1, n_{\Delta'_L} = 1, n_{\delta_L^c} = 1, n_{\delta_L'^c} = -1, n_{\Phi} = n_{\Phi'} = 0, \\
n_L = \frac{1}{2}, n_{L^c} = -\frac{1}{2}, n_Q = \frac{1}{2}, n_{Q^c} = -\frac{1}{2},
\end{aligned} \tag{8}$$

the chiral superfields of this model will transform as

$$\begin{aligned}
\hat{\Delta}_L(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\Delta}_L(x, \theta, \bar{\theta}), \hat{\Delta}'_L(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\Delta}'_L(x, \theta, \bar{\theta}), \hat{\delta}_L^c(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\delta}_L^c(x, \theta, \bar{\theta}), \hat{\delta}_L'^c(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\delta}_L'^c(x, \theta, \bar{\theta}), \\
\hat{\Phi}(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\Phi}(x, \theta, \bar{\theta}), \hat{\Phi}'(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\Phi}'(x, \theta, \bar{\theta}), \hat{L}(x, \theta, \bar{\theta}) \xrightarrow{R_d} -\hat{L}(x, \theta, \bar{\theta}), \hat{L}^c(x, \theta, \bar{\theta}) \xrightarrow{R_d} -\hat{L}^c(x, \theta, \bar{\theta}), \\
\hat{Q}(x, \theta, \bar{\theta}) \xrightarrow{R_d} -\hat{Q}(x, \theta, \bar{\theta}), \hat{Q}^c(x, \theta, \bar{\theta}) \xrightarrow{R_d} -\hat{Q}^c(x, \theta, \bar{\theta}),
\end{aligned} \tag{9}$$

In terms of the field components, we obtain

$$\begin{aligned}
H(x) \xrightarrow{R_d} H(x), \quad \tilde{f}(x) \xrightarrow{R_d} -\tilde{f}(x), \\
\tilde{H}(x) \xrightarrow{R_d} -\tilde{H}(x), \quad \Psi(x) \xrightarrow{R_d} \Psi(x).
\end{aligned} \tag{10}$$

where H is a usual scalar, \tilde{H} is the higgsinos, while Ψ is the fermion and \tilde{f} is a sfermion.

Equation (10) suggests to classify the particles into two types of so called R-even and R-odd. Here the R-even particles ($R_d = +1$) include the quarks, the leptons and the Higgs bosons. Whereas, the R-odd particles ($R_d = -1$) are their superpartners, i.e., neutralinos, charginos, squarks and sleptons. Therefore, R-parity is parity of R-charge of the continuous $U(1)$ R-symmetry and defined by

$$\text{R-parity} = \begin{cases} +1 & \text{for ordinary particles,} \\ -1 & \text{for their superpartners.} \end{cases} \tag{11}$$

The above intimate connection between R-parity and baryon number, lepton number conservation laws can be made explicitly by re-expressing (11) in terms of the spin S and the matter-parity $(-1)^{3(B-L)}$ as follows [?]:

$$\text{R-parity} = (-1)^{2S}(-1)^{3(B-L)}. \quad (12)$$

Therefore, all scalar fields ($S = 0$) can be assigned R values

- Usual scalars: $B = L = 0 \implies R = +1$,
- Sleptons: $B = 0, L = 1 \implies R = -1$,
- Squarks: $B = \frac{1}{3}, L = 0 \implies R = -1$,

Analogously for fermions ($S = 1/2$)

- Higgsinos $B = 0, L = 0 \implies R = -1$,
- Leptons: $B = 0, L = 1 \implies R = +1$,
- Quarks: $B = \frac{1}{3}, L = 0 \implies R = +1$.

To finish this section, let us note that there will be a lot of other choices of the charges due to the action of R-symmetry which in a general way can be written as [?]

$$\Phi \longrightarrow e^{2in_\Phi \frac{2\pi}{N}} \Phi. \quad (13)$$

Here it is similar to a Z_N symmetry. Among those choices, there is a possibility which is known as Lepton Parity where we choose

$$\begin{aligned} n_{\Delta_L} = 0, n_{\Delta'_L} = 0, n_{\delta_L^c} = 0, n_{\delta'_L} = 0, n_\Phi = n_{\Phi'} = 0, \\ n_L = \frac{1}{2}, n_{L^c} = -\frac{1}{2}, n_Q = 0, n_{Q^c} = 0. \end{aligned} \quad (14)$$

The reason to this name is due the fact that we get the following transformation of the superfields

$$(\hat{L}, \hat{L}^c) \rightarrow -(\hat{L}, \hat{L}^c). \quad (15)$$

while all others fields are even. All the terms in the superpotential, see Eq.(6), are allowed by the Lepton Parity. It is important to say that in this case the formula $(-1)^{3(B-L)}$ does not hold on this case.

Other, interesting, possibility is the Baryon Parity. On this case we want this transformation

$$(\hat{Q}, \hat{Q}^c) \rightarrow -(\hat{Q}, \hat{Q}^c). \quad (16)$$

while all others fields are even. To get this transformation law, we need to choose this charges

$$\begin{aligned} n_{\Delta_L} = 0, n_{\Delta'_L} = 0, n_{\delta_L^c} = 0, n_{\delta'_L} = 0, n_\Phi = n_{\Phi'} = 0, \\ n_L = 0, n_{L^c} = 0, n_Q = \frac{1}{2}, n_{Q^c} = -\frac{1}{2}. \end{aligned} \quad (17)$$

Superfield	Usual Particle	Spin	Superpartner	Spin
$\hat{V}' (U(1))$	B_m	1	\tilde{B}	$\frac{1}{2}$
$\hat{V}_L^i (SU(2)_L)$	W_{mL}^i	1	\tilde{W}_L^i	$\frac{1}{2}$
$\hat{V}_R^i (SU(2)_R)$	W_{mR}^i	1	\tilde{W}_R^i	$\frac{1}{2}$
$\hat{V}_c^a (SU(3))$	g_m^a	1	\tilde{g}^a	$\frac{1}{2}$
$\hat{Q}_i \sim (\mathbf{3}, \mathbf{2}, \mathbf{1}, 1/3)$	$(u_i, d_i)_{iL}$	$\frac{1}{2}$	$(\tilde{u}_{iL}, \tilde{d}_{iL})$	0
$\hat{Q}_i^c \sim (\mathbf{3}^*, \mathbf{1}, \mathbf{2}, -1/3)$	$(d_i^c, -u_i^c)_{iL}$	$\frac{1}{2}$	$(\tilde{d}_{iL}^c, -\tilde{u}_{iL}^c)$	0
$\hat{L}_a \sim (\mathbf{1}, \mathbf{2}, \mathbf{1}, -1)$	$(\nu_a, l_a)_{aL}$	$\frac{1}{2}$	$(\tilde{\nu}_{aL}, \tilde{l}_{aL})$	0
$\hat{L}_a^c \sim (\mathbf{1}, \mathbf{1}, \mathbf{2}, 1)$	$(l_a^c, -\nu_a^c)_{aL}$	$\frac{1}{2}$	$(\tilde{l}_{aL}^c, -\tilde{\nu}_{aL}^c)$	0
$\hat{\chi}_{1L} \sim (\mathbf{1}, \mathbf{2}, \mathbf{1}, 1)$	$(\chi_{1L}^+, \chi_{1L}^0)$	0	$(\tilde{\chi}_{1L}^+, \tilde{\chi}_{1L}^0)$	$\frac{1}{2}$
$\hat{\chi}_{2L} \sim (\mathbf{1}, \mathbf{2}, \mathbf{1}, -1)$	$(\chi_{2L}^0, \chi_{2L}^-)$	0	$(\tilde{\chi}_{2L}^0, \tilde{\chi}_{2L}^-)$	$\frac{1}{2}$
$\hat{\chi}_{3L}^c \sim (\mathbf{1}, \mathbf{1}, \mathbf{2}, -1)$	$(\chi_{3L}^0, \chi_{3L}^-)$	0	$(\tilde{\chi}_{3L}^0, \tilde{\chi}_{3L}^-)$	$\frac{1}{2}$
$\hat{\chi}_{4L}^c \sim (\mathbf{1}, \mathbf{1}, \mathbf{2}, 1)$	$(\chi_{4L}^+, \chi_{4L}^0)$	0	$(\tilde{\chi}_{4L}^+, \tilde{\chi}_{4L}^0)$	$\frac{1}{2}$
$\hat{\Phi} \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0)$	$\begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$	0	$\begin{pmatrix} \tilde{\phi}_1^0 & \tilde{\phi}_1^+ \\ \tilde{\phi}_2^- & \tilde{\phi}_2^0 \end{pmatrix}$	$\frac{1}{2}$
$\hat{\Phi}' \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}, 0)$	$\begin{pmatrix} \chi_1^0 & \chi_1^+ \\ \chi_2^- & \chi_2^0 \end{pmatrix}$	0	$\begin{pmatrix} \tilde{\chi}_1^0 & \tilde{\chi}_1^+ \\ \tilde{\chi}_2^- & \tilde{\chi}_2^0 \end{pmatrix}$	$\frac{1}{2}$

These Baryon Parity doesn't eliminate any terms in our superpotential. As in the previous case, we can not write the R charge as $(-1)^{3(B-L)}$.

In the case of the MSSM we choose the Lepton Parity, defined at Eq.(14), and the Baryon Parity, see Eq.(17), in order to avoid the proton decay. It happens because using these "new" parity we can eliminate some terms in the superpotential of the MSSM ^{?,?,?}.

However it is not the case on this model. The only difference between the traditional R -parity and these parities defined above is that now the Eq.(12) is not satisfied.

3. SUSYLRD

This model contains the particle content given at Tab.(2). The Lagrangian of this model is given at Ref. ¹⁸.

The most general superpotential and soft supersymmetry breaking Lagrangian

for this model are:

$$\begin{aligned}
W = & M_{\chi} \hat{\chi}_1 \hat{\chi}_2 + M_{\chi^c} \hat{\chi}_3^c \hat{\chi}_4^c + \mu_1 Tr(\tau_2 \hat{\Phi} \tau_2 \hat{\Phi}) + \mu_2 Tr(\tau_2 \hat{\Phi}' \tau_2 \hat{\Phi}') + \mu_3 Tr(\tau_2 \hat{\Phi} \tau_2 \hat{\Phi}') \\
& + h_{ab}^l Tr(\hat{L}_a \hat{\Phi} \tau_2 \hat{L}_b^c) + \tilde{h}_{ab}^l Tr(\hat{L}_a \hat{\Phi}' \tau_2 \hat{L}_b^c) + h_{ij}^q Tr(\hat{Q}_i \hat{\Phi} \tau_2 \hat{Q}_j^c) \\
& + \tilde{h}_{ij}^q Tr(\hat{Q}_i \hat{\Phi}' \tau_2 \hat{Q}_j^c) + W_{NR}.
\end{aligned} \tag{18}$$

Applying the invariance conditions under R -parity transformation in Eq.(18) we get the following equations

$$\begin{aligned}
n_{\chi_1} + n_{\chi_2} = 0, n_{\chi_3^c} + n_{\chi_4^c} = 0, 2n_{\Phi} = 2n_{\Phi'} = 0, n_{\Phi} + n_{\Phi'} = 0, 2n_L + n_{\Delta_L} = 0, 2n_{L^c} + n_{\delta_L^c} = 0, \\
n_L + n_{L^c} + n_{\Phi} = 0, n_L + n_{L^c} + n_{\Phi'} = 0, n_Q + n_{Q^c} + n_{\Phi} = 0, n_Q + n_{Q^c} + n_{\Phi'} = 0..
\end{aligned} \tag{19}$$

For example, choosing

$$\begin{aligned}
n_{\chi_1} = 0, n_{\chi_2} = 0, n_{\chi_3^c} = 0, n_{\chi_4^c} = 0, n_{\Phi} = n_{\Phi'} = 0, \\
n_L = \frac{1}{2}, n_{L^c} = -\frac{1}{2}, n_Q = \frac{1}{2}, n_{Q^c} = -\frac{1}{2},
\end{aligned} \tag{20}$$

the chiral superfields of this model will transform as

$$\hat{\chi}_1(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\chi}_1(x, \theta, \bar{\theta}), \hat{\chi}_2(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\chi}_2(x, \theta, \bar{\theta}), \hat{\chi}_3^c(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\chi}_3^c(x, \theta, \bar{\theta}), \hat{\chi}_4^c(x, \theta, \bar{\theta}) \xrightarrow{R_d} \hat{\chi}_4^c(x, \theta, \bar{\theta}), \tag{21}$$

With this charges we reproduce Eqs.(10, 12).

As happened on the first model analyzed above, if we define the L -parity and the B -parity we can not again eliminate any term in the superpotential defined at Eq.(18) and the same comments make at that time are still hold on this case.

4. Conclusion

We have showed explicitly that the superpotential of the models SUSYLRT and SUSYLTD they satisfy the R -parity, and in this case its values are given by $R = (-1)^{3(B-L)+2S}$. In contrast with the MSSM case on this kind of model the R -parity does not eliminate any terms in the superpotential.

We show also that they also satisfy the lepton and baryon parity. On this case we can not use the expression $(-1)^{3(B-L)}$ to define the R -parity.

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